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University of Wisconsin–Madison
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Introduction

This is the final report of research performed under Contract DASG60-91-C-0144. It covers major areas of research, publications and technical papers, participating scientific personnel, degrees awarded, reportable inventions (negative report), and a chronological account of the work undertaken.

Detailed coverage of the research undertaken is provided in the extracts included here from the applicable quarterly reports for each quarter after the startup period. In some cases these reports make reference to attachments (for example drafts of papers, copies of viewgraph slides, etc.). These have been withdrawn.

1. Major areas of research

Cost-benefit analysis and marginal value analysis. Quantitative methods for comparing cost vs. benefit and for estimating marginal value of weapon and other systems comprising military forces (Publications 1,2,3).

This area includes problems of cost/benefit analysis where instead of the classical single criterion (with associated costs and benefits) there are several criteria that are *a priori* incommensurable. One wishes to allocate limited resources to a portfolio of resource-users (for example, projects), each of which has certain specified costs and benefits on each of the criteria. This could, for example, describe situations in the management of research and development, or in allocation of resources during battle management.

A well known method that people have applied to such problems in the past is the Analytic Hierarchy Process (AHP) devised by T. Saaty. This method is in use in numerous areas, and has been implemented in a popular software package *(Expert Choice).* The AHP applies to a problem somewhat less general than that described above, in which each user (project) is associated with just one criterion. It produces a set of prices, or weights, indicating the relative importance of the different projects. However, a severe drawback of the AHP has been that these prices are produced by a "black-box" mathematical procedure that does not offer any interpretation. Therefore, it has not been clear that these prices measured any real aspect of resource tradeoffs among the different projects.

A central result that we obtained was that for the general model treated – which includes the AHP as a special case – the prices produced by the model represented marginal rates of substitution for the cost/benefit index used in the model. Therefore these are in fact real prices. Of course, this conclusion applies in particular to the AHP, and it shows that the output of the AHP does indeed have an economic interpretation, as people had suspected but had not been able to prove.

Scenario analysis. Methods for obtaining optimal "hedged" decisions under scenario representations of an uncertain future (Publication 7).

Scenario analysis problems arise in resource allocation decisions containing uncertainties that can be modeled by finite probability distributions. They have a wide variety of applications; one class of applications, investigated under a separate contract with the

US Army Research Office, consists of so-called "mix analysis" problems, in which one asks what is the best choice of weapon systems for a tactical unit (for example, an Army brigade) which will have to fight in a variety of different environments against different types of enemy forces. By applying a probability distribution to the array of force types and environments, one can formulate the problem of finding a mix of equipment yielding best expected performance, under various kinds of constraints (such as budget limits, ramp-up or ramp-down constraints, limits on allowable numbers of personnel, constraints on ammunition or on operating costs, etc.).

Using various model types for testing, including stochastic networks and equipment mix problems, we succeeded in improving substantially on the performance of current methods for solving the resulting large deterministic programming problems. The best current technology is the so-called "progressive hedging algorithm," developed by Rockafellar and Wets. Experiments indicate that this method can converge extremely slowly. In addition, the method contains parameters that can substantially influence the rate of convergence of the method; however, there is no way to determine in advance the proper values to use for these parameters.

By using decomposition methods based on the bundle trust region (BTR) algorithm, developed by Schramm and Zowe, we substantially improved on the computing times needed by the progressive hedging algorithm. The bundle decomposition method that we developed solved stochastic optimization problems in times that were from 3 to 1563 times faster than those obtained from the progressive hedging algorithm with the parameter value $r = 1$. The weighted average speedup factor was 14.4. Therefore it appears that our method results in a very substantial saving of time for the type of problem we investigated (problems with moderate numbers of coupling constraints).

Optimization of systems described by simulation. Finding optimal parameter values under uncertainty for systems, such as production lines and project-management networks, whose response cna be studied only by computer simulation (Publications 5, 6, 9, 10).

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to

use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow, particularly so when there are constraints on the allowable values of the parameters (for example, upper or lower bounds). Methods based on stochastic approximation may also experience difficulties if the function being optimized is not differentiable at certain points.

A major focus of work under this contract was the application to simulation optimization of modern algorithms from nonsmooth analysis (specifically, methods of the bundle type), often in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination methods are intended to replace the method of stochastic approximation, which is the standard method used in this area to date.

A very significant advantage of the bundle-type methods is that they can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause severe difficulty for methods of stochastic approximation. Therefore, development and application of these methods should significantly expand the applicability and usefulness of this kind of stochastic optimization. This work was done in cooperation with the research group of Professor Rajan Suri at the University of Wisconsin–Madison.

Nonsmooth analysis and optimization. The study of how to optimize functions that are not necessarily differentiable but may have other desirable properties, such as convexity; a foundation area supporting the other areas described above (Publications 4, 8).

This theoretical foundation is necessary in order to make technical progress in the applied areas cited, since those areas involve construction of algorithms and analytic methods

for problems that cannot be handled with current methods. Most of this work was funded primarily under other research programs, but as explained above there is a significant connection to work under this contract.

In addition, other areas were studied during the course of work on this contract, including global optimization and estimation of the value of information in testing. However, these have not as yet resulted in publications in the open literature.

2. Publications and technical reports

The following papers acknowledge support from Contract DASG60-91-C-0144:

1. S. M. Robinson, "Shadow prices for measures of effectiveness, Part I: Linear Model," *Operations Research* **41** (1993) 518–535.

2. S. M. Robinson, "Shadow prices for measures of effectiveness, Part II: General Model," *Operations Research* **41** (1993) 536–548.

3. S. M. Robinson, "Minimax cost-benefit analysis," submitted to *Management Science*.

4. S. M. Robinson, "Nonsingularity and symmetry for linear normal maps," *Mathematical Programmming* **62** (1993) 415–425.

5. S. M. Robinson, "Convergence of subdifferentials under strong stochastic convexity," accepted by *Management Science*.

6. E. L. Plambeck, B.-R. Fu, S. M. Robinson, and R. Suri, "Throughput optimization in tandem production lines via nonsmooth programming," *Proceedings of 1993 Summer Computer Simulation Conference*, Society for Computer Simulation, San Diego, CA 1993, pp. 70–75.

7. B. J. Chun and S. M. Robinson, "Scenario analysis via bundle decomposition," submitted to *Annals of Operations Research*.

8. S. M. Robinson, "Newton's method for a class of nonsmooth functions," accepted by *Set-Valued Analysis*.

9. E. L. Plambeck, B.-R. Fu, S. M. Robinson, and R. Suri, "Sample-path optimization of convex stochastic performance functions," submitted to *Mathematical Programming*.

10. S. M. Robinson, "Analysis of sample-path optimization," submitted to *Management Science*.

3. Participating scientific personnel; degrees awarded

The following scientific personnel participated in the work under Contract DASG60-91-C-0144 during part or all of its duration.

Stephen M. Robinson, Professor.

Bradbury Franklin, Research Assistant.

Erica L. Plambeck, Undergraduate Assistant (Ms. Plambeck qualified for the degree of Bachelor of Science – Industrial Engineering. The degree was awarded in May 1994).

4. Reportable inventions

To the best of the principal investigator's knowledge, there were no reportable inventions during the course of this research.

5. Chronological account of research performed

Quarter 2, Calendar Year 1992

Summary of Research Progress

This summary covers research activities for the cited contract during the second quarter of CY 1992. It discusses work in three major areas:

1. Numerical methods for scenario analysis (optimization of resource allocation under uncertainty).
2. Using stochastic optimization in decision analysis.
3. Optimization of simulations, with application to manufacturing problems.

a. *Numerical methods for scenario analysis.*

A principal area of activity during this quarter was the solution of large linear or quadratic stochastic optimization problems by decomposition. As was discussed in last quarter's report, such problems arise in resource allocation decisions containing uncertainties that can be modeled by finite probability distributions. They have a wide variety of applications; one class of applications, being investigated under a separate contract with the US Army Research Office, consists of so-called "mix analysis" problems, in which one asks what is the best choice of weapon systems for a tactical unit (for example, an Army brigade) which will have to fight in a variety of different environments against different types of enemy forces. By applying a probability distribution to the array of force types and environments, one can formulate the problem of finding a mix of equipment yielding best expected performance, under various kinds of constraints (such as budget limits, ramp-up or ramp-down constraints, limits on allowable numbers of personnel, constraints on ammunition or on operating costs, etc.).

Using various model types for testing, including stochastic networks and equipment mix problems, we have succeeded in improving substantially on the performance of current methods for solving the resulting large deterministic programming problems. The best current technology is the so-called "progressive hedging algorithm," developed by Rockafellar and Wets. Experiments indicate that this method can converge extremely slowly.

In addition, the method contains parameters that can substantially influence the rate of convergence of the method: however, there is no way to determine in advance the proper values to use for these parameters.

By using decomposition methods based on the bundle trust region (BTR) algorithm of Schramm and Zowe, we have substantially improved on the computing times needed by the progressive hedging algorithm. Results of tests comparing these methods were obtained and presented to the 4th Conference on Optimization of the Society for Industrial and Applied Mathematics (SIAM), held in Chicago, 11-13 May 1992. Copies of the viewgraphs presented at this meeting were furnished to the COTR (Mr. Pathak), and a copy is also attached as Enclosure 1 to this report.

As is shown in more detail in Enclosure 1, the bundle decomposition method that we have developed solved stochastic optimization problems in times that were from 3 to 1563 times faster than those obtained from the progressive hedging algorithm with the parameter value $r = 1$. The weighted average speedup factor was 14.4. Therefore it appears that our method results in a very substantial saving of time for the type of problem we are investigating (problems with moderate numbers of coupling constraints). We are currently planning preparation of one or more journal articles describing this work (see section on Planned Activities below).

b. Using stochastic optimization in decision analysis.

The second major area of work this quarter was in the integration of stochastic optimization into decision analysis. The idea is to employ, in place of the conventional decision tree, a stochastic optimization model that handles the task of selecting all or part of an optimal decision. Use of such a model makes it possible to deal, in a decision analysis framework, with very complex optimization models involving hundreds or thousands of variables. Such situations could occur, for example, in modeling of proposed weapon systems where one wished to decide whether to deploy a proposed system or to conduct further testing. Using the framework of Bayesian preposterior analysis along with information about the accuracy of the proposed tests and their cost, one could use simulations of the system's performance to estimate the value added by further testing and, consequently, to judge whether such testing was worthwhile.

This line of work was originally developed for the Deputy Under Secretary of the Army (Operations Research) [DUSA(OR)], and it has led to a draft paper explaining the method and illustrating it with a simple example involving tests of a weapon system. A copy of the draft as of June 1992 is attached as Enclosure 2.

Before submitting this work for publication we would like to gain some experience with the method in tests on an actual system, so that the results of this practical application could be reported in the paper and thus could reinforce the theoretical development already completed. We are attempting to obtain access to test problems through the DUSA(OR), either from the Army Materiel Systems Analysis Activity (AMSAA) or from the Test and Experimentation Command. Therefore, we expect to delay journal submission of this work until we can determine whether such test examples will be available and, if so, apply the method to them. Contacts for the purpose of obtaining test examples are expected to be made during the coming calendar quarter (July - September 1992).

c. Optimization of simulations, with application to manufacturing systems.

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow.

Parameter optimization in such simulations leads to stochastic optimization problems that have some elements in common with the scenario analysis problems discussed in (a) above. As methods based on the bundle/trust region algorithm have turned out to be very effective for scenario analysis, we are deriving such methods for the problem of optimizing

simulations and will be trying them out this coming summer on simple problems from manufacturing. This work will be done in cooperation with the research group of Professor Rajan Suri at Madison.

During this quarter we began the programming and experimentation effort, with one assistant assigned to work full time during the summer on this project. Results by the end of September will be described in the next quarterly report.

Quarterly Review

A quarterly review was held on 12 June 1992 at the University of Wisconsin–Madison. Copies of viewgraphs prepared to support that review are attached as Enclosure 3.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work in the three areas already described and to begin a fourth major area.

a. *Numerical methods for scenario analysis.*

We are currently planning preparation of one or more journal articles describing the work performed in stochastic optimization of scenario analysis problems. During the coming quarter we expect to complete the planning and to outline and partially write the resulting papers. It is not likely that the writing will proceed quickly enough for the papers actually to be submitted to journals by the end of September.

b. *Using stochastic optimization in decision analysis.*

During the coming quarter we expect to make contacts with Army agencies for the purpose of obtaining test examples for the paper on "Decision Analysis Via Stochastic Optimization" (Enclosure 2). This will probably include presenting briefings at one or more agencies to acquaint them with the material. It is possible that actual progress on testing may be made during the quarter, but this is not clear now.

c. *Optimization of simulations, with application to manufacturing systems.*

During the forthcoming quarter we plan to test the bundle decomposition algorithm on problems from manufacturing, beginning with a simple problem of optimizing flow rates of machines in a transfer line. This work will be done in cooperation with the research

group of Professor Rajan Suri at Madison. Based on the results obtained from this testing, we will decide whether to continue and expand this effort.

d. Global optimization and stochastic differential equations. During the third quarter of CY 92 we plan to begin an effort to develop improved methods for global optimization of unconstrained and, later, constrained nonlinear functions. The general approach we envision is based on following trajectories of ordinary differential equations that have been altered by inclusion of a scaled Wiener process (Brownian motion). In the coming quarter we expect to be able to begin surveying previous work in the area and acquiring the necessary tools in stochastic differential equations. This work is likely to be a long-term project, and we are just at the beginning. Therefore we do not anticipate significant accomplishments in this area by September.

Quarter 3, Calendar Year 1992

Summary of Research Progress

This summary covers research activities for the cited contract during the third quarter of CY 1992. It discusses work in four major areas:

a. Cost/benefit analysis and decision analysis, with application to problems involving multiple criteria and/or objectives.

b. Optimization of simulations, with application to manufacturing problems.

c. Nonsmooth analysis with application to optimization and variational problems.

d. Stochastic methods for global optimization of nonconvex functions (initial work only).

a. *Cost/benefit analysis and decision analysis, with application to problems involving multiple criteria and/or objectives.*

A significant achievement made during this quarter was the preparation of a paper entitled, "Minimax cost/benefit analysis." This paper describes a framework for doing cost/benefit analysis where instead of the classical single criterion (with associated costs and benefits) there are several criteria that are *a priori* incommensurable. One wishes to allocate limited resources to a portfolio of resource-users (for example, projects), each of which has certain specified costs and benefits on each of the criteria. This could, for example, describe situations in the management of research and development, or in allocation of resources during battle management.

A well known method that people have applied to such problems in the past is the Analytic Hierarchy Process (AHP) devised by T. Saaty. This method is in use in numerous areas, and has been implemented in a popular software package *(Expert Choice)*. The AHP applies to a problem somewhat less general than that described above, in which each user (project) is associated with just one criterion. It produces a set of prices, or weights, indicating the relative importance of the different projects. However, a severe drawback of the AHP has been that these prices are produced by a "balck-box" mathematical procedure

that does not offer any interpretation. Therefore, it has not been clear that these prices measured any real aspect of resource tradeoffs among the different projects.

A central result of the paper mentioned above was that for the general model it treated – which includes the AHP as a special case – the prices produced by the model represented marginal rates of substitution for the cost/benefit index used in the model. Therefore these are in fact real prices. Of course, this conclusion applies in particular to the AHP, and it shows that the output of the AHP does indeed have an economic interpretation, as people had suspected but had not been able to prove.

The paper describing this cost/benefit model is attached as Enclosure 1. It has been submitted to *Management Science*.

Work also continued this quarter on the integration of stochastic optimization into decision analysis. The idea is to employ, in place of the conventional decision tree, a stochastic optimization model that handles the task of selecting all or part of an optimal decision. Use of such a model makes it possible to deal, in a decision analysis framework, with very complex optimization models involving hundreds or thousands of variables. Such situations could occur, for example, in modeling of proposed weapon systems where one wished to decide whether to deploy a proposed system or to conduct further testing. Using the framework of Bayesian preposterior analysis along with information about the accuracy of the proposed tests and their cost, one could use simulations of the system's performance to estimate the value added by further testing and, consequently, to judge whether such testing was worthwhile.

During this quarter we discussed this work with analysts and managers at the U.S. Army Materiel Systems Analysis Agency (AMSAA), Aberdeen Proving Ground, MD. In particular, we were interested in the possibility of testing this method on an actual system, so that the results of this practical application could be reported in the paper and thus could reinforce the theoretical development already completed.

AMSAA personnel were interested in the technique, and they indicated that they would be willing to cooperate in future testing on actual systems. However, they pointed out a significant difficulty in the application of the method: in many cases, a utility function different from expected monetary value will be needed in order to reflect the risk

attitudes and preferences of the decision maker. However, in Army system development efforts there is no identifiable decision maker whose utility function could be assessed. Therefore, it is likely to be necessary to construct some kind of "justification function" or other means of reflecting the performance and cost attributes desired in a system, when these are uncertain. It is by no means clear how this should be done, but we plan to continue work on this topic.

b. Optimization of simulations, with application to manufacturing problems.

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow.

Parameter optimization in such simulations leads to stochastic optimization problems that have some elements in common with the scenario analysis problems previously investigated under this research program. As methods based on the bundle/trust region algorithm for nonsmooth optimization proved to be very effective for scenario analysis, we have derived such methods for the problem of optimizing simulations. This work is being done in cooperation with the research group of Professor Rajan Suri at Madison.

During this quarter we obtained computational results for the optimization of expected throughput of a tandem manufacturing line, with the decision variables being the work flow rates assigned to the various machines. Some results from these experiments were presented by invitation at the VI International Conference on Stochastic Programming, held at the International Center for Mechanical Sciences, Udine, Italy, in September 1992.

This lecture also included results from the earlier work on scenario analysis. A copy of the slides for this presentation is attached as Enclosure 2.

A demonstration of an optimization run was prepared and presented to the COTR (Mr. K. Pathak) during the quarterly review held at Madison on 4 September 1992. Work in this area is continuing at a substantial level, and we anticipate that a first paper should be completed and submitted for publication during the coming calendar quarter (October – December 1992).

c. *Nonsmooth analysis with application to optimization and variational problems.*

This is a supporting technical area whose results are applicable in more applied areas such as those described in (a) and (b) above. It involves development of mathematical properties of functions appearing in optimization problems, which are not differentiable but may have other good properties such as convexity. This theoretical foundation is necessary in order to make technical progress in the applied areas cited, since those areas involve construction of algorithms and analytic methods for problems that cannot be handled with current methods. Most of this work is funded under other research programs, but as explained above there is a significant connection to work under this contract.

During this quarter we completed and submitted to *Mathematical Programming* the revised version of a paper describing the solution of a basic problem in this area: namely, the characterization of unique, continuous solvability of a symmetric linear variational inequality on a polyhedral convex set. A copy of this paper is attached as Enclosure 3.

d. *Stochastic methods for global optimization of nonconvex functions (initial work only).*

During this quarter we began work on developing computational methods for optimizing functions that are not necessarily convex. Good methods for local optimization of such functions are already available, but finding a global optimum is an unsolved (and very hard) problem. We have begun to investigate methods based on the use of stochastic differential equations, whose solutions evolve over time in such a way that for large time values the solution is a random variable whose probability distribution is heavily concentrated in the region(s) of the global minimizer(s). In this quarter only preliminary work was done to get the research effort underway. Some computational results of the initial im-

plementations and experiments (with variants of known methods) are likely to be available during the coming quarter.

Quarterly Review

A quarterly review was held on 4 September 1992 at the University of Wisconsin-Madison, by Mr. K. Pathak, COTR. This review consisted primarily of discussion of the work done in different areas with computer demonstration of techniques. Viewgraphs were not necessary. It is expected that the next review will be held at Huntsville, AL.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work in the four principal areas described above. The last area (stochastic methods for global optimization) is likely to comprise a large share of the effort, together with the work on optimization of simulations.

Quarter 4, Calendar Year 1992

Summary of Research Progress

This summary covers research activities for the cited contract during the fourth quarter of CY 1992. It discusses work in three major areas:

a. Optimization of simulations, with application to manufacturing problems.

b. Nonsmooth analysis with application to optimization and variational problems.

c. Stochastic methods for global optimization of nonconvex functions (initial work only).

a. *Optimization of simulations, with application to manufacturing problems.*

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow.

Parameter optimization in such simulations leads to stochastic optimization problems that have some elements in common with the scenario analysis problems previously investigated under this research program. As methods based on the bundle/trust region algorithm for nonsmooth optimization proved to be very effective for scenario analysis, we have derived such methods for the problem of optimizing simulations. This work is being done in cooperation with the research group of Professor Rajan Suri at Madison.

During this quarter we continued work on the development and computer implementation of optimization methods that use modern algorithms from nonsmooth analysis (specifically, the Bundle/Trust Region (BTR) method of Schramm and Zowe) in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination methods are intended to replace the method of stochastic approximation, which is the standard method used in this area to date. The advantage of the BTR/IPA method is that it can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause great difficulty for methods of stochastic approximation. Therefore, the development of this method should expand significantly the applicability and usefulness of this kind of stochastic optimization.

The main work this quarter focused on enhancing the interface between the simulation code used to obtain the IPA estimates, and the BTR optimization code, with particular attention to cases in which the objective is nonsmooth. This nonsmoothness can occur, for example, in the long-run average throughput of a tandem manufacturing line when the flow rates of one or more machines in the line become equal.

As part of this work, we began preparation of a paper extending the standard "strong consistency" justification of IPA to the nonsmooth case. This paper is expected to be completed and submitted for publication in the next calendar quarter.

b. Nonsmooth analysis with application to optimization and variational problems.

This is a supporting technical area whose results are applicable in more applied areas such as those described in (a) and (b) above. It involves development of mathematical properties of functions appearing in optimization problems, which are not differentiable but may have other good properties such as convexity. This theoretical foundation is necessary in order to make technical progress in the applied areas cited. since those areas involve construction of algorithms and analytic methods for problems that cannot be handled with current methods. Most of this work is funded under other research programs, but as explained above there is a significant connection to work under this contract.

During this quarter we completed and submitted to *Annals of Operations Research*

a long paper showing how the BTR method described in the last section could be used to solve large-scale linear programming problems resulting from the technique of scenario analysis for stochastic optimization. The BTR method resulted in solutions of scenario analysis problems up to 1,500 times faster than the major competitive technique for large problems, namely the "progressive hedging algorithm." Extensive computational results were presented to compare the two techniques. A copy of this paper is attached as Enclosure 2.

c. *Stochastic methods for global optimization of nonconvex functions.*

During this quarter we continued work on developing computational methods for optimizing functions that are not necessarily convex. Good methods for local optimization of such functions are already available, but finding a global optimum is an unsolved (and very hard) problem. We are investigating methods based on the use of stochastic differential equations, whose solutions evolve over time in such a way that for large time values the solution is a random variable whose probability distribution is heavily concentrated in the region(s) of the global minimizer(s).

In this quarter we implemented some computational methods from the literature and tested them on several hard problems of global optimization. Some results of the initial implementations and experiments were presented during the December 1992 quarterly review (see below). Work in this area is expected to continue during the remainder of the contract; the area is very hard, and at this time we cannot predict how successful it will be. However, we believe that our approach offers the best chances of success of any known to us.

Quarterly Review

A quarterly review was held on 16 December 1992 at the U.S. Army Space and Strategic Defense Command, Huntsville, AL. by Mr. K. Pathak, COTR. A copy of the viewgraphs for this review is attached as Enclosure 1.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work in the three principal areas described above. The last area (stochastic methods for global optimization) is likely to comprise a large share of the effort, together with the work on optimization of simulations.

Quarter 1, Calendar Year 1993

Summary of Research Progress

This summary covers research activities for the cited contract during the first quarter of CY 1993. It discusses work in three major areas:

a. Optimization of simulations, with application to manufacturing problems.

b. Stochastic methods for global optimization of nonconvex functions.

c. Measuring value of information, with application to system testing.

a. *Optimization of simulations, with application to manufacturing problems.*

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However. if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow.

Parameter optimization in such simulations leads to stochastic optimization problems that have some elements in common with the scenario analysis problems previously investigated under this research program. As methods based on the bundle/trust region algorithm for nonsmooth optimization proved to be very effective for scenario analysis, we have derived such methods for the problem of optimizing simulations. This work is being done in cooperation with the research group of Professor Rajan Suri at Madison.

During this quarter we continued work on the development and computer implementation of optimization methods that use modern algorithms from nonsmooth analysis

(specifically, the Bundle/Trust Region (BTR) method of Schramm and Zowe) in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination methods are intended to replace the method of stochastic approximation, which is the standard method used in this area to date. The advantage of the BTR/IPA method is that it can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause great difficulty for methods of stochastic approximation. Therefore, the development of this method should expand significantly the applicability and usefulness of this kind of stochastic optimization.

The main work this quarter focused on completing and submitting for publication a paper extending the standard "strong consistency" justification of IPA to the nonsmooth case. This paper, entitled "Convergence of subdifferentials under strong stochastic convexity," was submitted to *Management Science*. A copy of this paper is attached as Enclosure 2.

Collateral work in this area included further computation of optimal parameter values for tandem production lines, using a simulation code to obtain the IPA estimates and the BTR method for optimization. This work was in support of a new paper explaining the methodology for this production-line application, for presentation at the Summer Computer Simulation Conference to be held in Boston, July 1993. This paper is expected to be completed during the next calendar quarter (April - June 1993).

We also began preliminary preparation for presentation of two invited lectures on the use of nonsmooth methods in stochastic optimization. These lectures are to be presented at the Joint National Meeting of The Institute of Management Sciences and the Operations Research Society of America in Chicago, IL, 17–19 May 1993.

b. Stochastic methods for global optimization of nonconvex functions.

During this quarter we continued work on developing computational methods for optimizing functions that are not necessarily convex. Good methods for local optimization of such functions are already available, but finding a global optimum is an unsolved (and very hard) problem. We are investigating methods based on the use of stochastic differential

equations, whose solutions evolve over time in such a way that for large time values the solution is a random variable whose probability distribution is heavily concentrated in the region(s) of the global minimizer(s).

In particular, we continued the implementation of computational methods from the literature and tests of those methods on several hard problems of global optimization. Results of this work were presented during the March 1993 quarterly review (see below). Work in this area is expected to continue during the remainder of the contract; the area is very hard, and at this time we cannot predict how successful it will be. However, we believe that our approach offers the best chances of success of any known to us.

c. *Measuring value of information, with application to system testing.*

In previous work we have developed methods for measuring the value of information in support of decisions on whether, and how much, to test items (for example, weapon systems) under development. The principal investigator met on 1 March 1993 at Aberdeen Proving Ground, MD, with representatives of the U.S. Army Materiel Systems Analysis Activity (AMSAA) to discuss possible applications of this methodology.

One application that may be suitable is the decision on whether to request a waiver by the Secretary of Defense on the legally required live fire testing of major systems. The law provides that a waiver may be granted provided that live fire testing would be "unreasonably expensive and impractical." The methodology appears to be applicable in estimating the value added by testing; this value could then be compared against the cost of testing to determine whether a live fire test would be "unreasonably expensive." Work on this area is planned to continue into the next calendar quarter.

Quarterly Review

A quarterly review was held on 22 March 1993 at the U.S. Army Space and Strategic Defense Command, Huntsville, AL. by Mr. K. Pathak, COTR. A copy of the viewgraphs for this review is attached as Enclosure 1.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work in the three principal areas described above. The first and last areas (optimization of simulations, with applications to manufacturing problems, and estimating the value of information) are likely to represent most of that quarter's effort.

Quarter 2, Calendar Year 1993

Summary of Research Progress

This summary covers research activities for the cited contract during the second quarter of CY 1993. It discusses work in three major areas:

a. Optimization of systems described by simulations, with application to manufacturing problems.

b. Stochastic methods for global optimization

c. Measuring value of information, with application to system testing.

a. Optimization of systems described by simulations

Area Overview

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow, particularly so when there are constraints on the allowable values of the parameters (for example, upper or lower bounds).

A major focus of work under this contract has been the application to simulation optimization of modern algorithms from nonsmooth analysis (specifically, methods of the bundle type) in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination methods

are intended to replace the method of stochastic approximation, which is the standard method used in this area to date.

A very significant advantage of the bundle-type methods is that they can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause great difficulty for methods of stochastic approximation. Therefore, development and application of these methods should significantly expand the applicability and usefulness of this kind of stochastic optimization. This work is being done in cooperation with the research group of Professor Rajan Suri at the University of Wisconsin–Madison.

Progress

The main accomplishment in this area during this quarter was the completion and submission for publication of a paper explaining the application of this methodology to a specific manufacturing problem (maximizing throughput in tandem production lines). This paper is scheduled for presentation at the 1993 Summer Computer Simulation Conference to be held in Boston, July 1993, and for publication in the proceedings of that meeting. A copy of this paper is attached as Enclosure 2.

We also prepared and presented two invited lectures on the use of nonsmooth methods in stochastic optimization, at the Joint National Meeting of The Institute of Management Sciences and the Operations Research Society of America in Chicago, IL, 17–19 May 1993. One lecture dealt with the stochastic optimization methodology described above, while the other covered earlier work on decomposition applied to static stochastic optimization of the scenario analysis type (described in earlier quarterly reports). These lectures attracted sizable audiences and generated requests for further information and for copies of the papers involved.

During this quarter we also began work on applying the above methodology to problems in stochastic scheduling, a hard and important area with many applications in such areas as manufacturing, project management, etc. Preliminary analysis was done on the stochastic PERT (Program Evaluation and Review Technique) problem. This is expected to be a major activity in the next quarter.

b. Stochastic methods for global optimization

Area Overview

This area involves development of computational methods for optimizing functions that are not necessarily convex. Good methods for local optimization of such functions are already available, but finding a global optimum is an unsolved (and very hard) problem. We are investigating methods based on the use of stochastic differential equations, whose solutions evolve over time in such a way that for large time values the solution is a random variable whose probability distribution is heavily concentrated in the region(s) of the global minimizer(s).

Progress

During this quarter we continued work on implementation of computational methods from the literature and tests of those methods on hard problems of global optimization. In particular, we focused on obtaining approximate global optimizers for the notorious Lennard-Jones 6-12 function, which models the potential energy in a molecule containing atoms of only one element. Results of this work were presented during the June 1993 quarterly review (see below). This work represented the final phase of planned investigation in this area; due to funding constraints we expect to allocate remaining money to higher-priority projects.

c. Measuring value of information

Area Overview

This area involves integrating stochastic optimization into decision analysis, and using the resulting set of tools to measure the value of information that could be obtained from projects or activities proposed for performance (for example, testing of weapon systems). The idea is to employ, in place of the conventional decision tree, a stochastic optimization model that handles the task of selecting all or part of an optimal decision. Use of such a model makes it possible to deal, in a decision analysis framework, with very complex optimization models involving hundreds or thousands of variables. Such situations could occur, for example, in modeling of proposed weapon systems where one wished to decide whether to deploy a proposed system or to conduct further testing.

Using the framework of Bayesian preposterior analysis along with information about the accuracy of the proposed tests and their cost, one can use simulations of the system's performance to estimate the value added by further testing and, consequently, to judge whether such testing is worthwhile.

Progress

Work continued this quarter on the problem, previously reported, of methodological support for a decision on whether to request a waiver by the Secretary of Defense on the legally required live fire testing of major systems. The law provides that a waiver may be granted provided that live fire testing would be "unreasonably expensive and impractical." The methodology appears to be applicable in estimating the value added by testing; this value could then be compared against the cost of testing to determine whether a live fire test would be "unreasonably expensive." Although a draft paper was prepared, further work is needed. It is possible that by the end of the next calendar quarter the work will have advanced far enough so that a finished paper may be ready.

Quarterly Review

A quarterly review was held on 1 June 1993 at the University of Wisconsin–Madison, Madison, WI, by Mr. K. Pathak, COTR. A copy of the viewgraphs for this review is attached as Enclosure 1.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work in the first and third of the principal areas described above (optimization of simulations, with applications to manufacturing problems, and estimating the value of information).

Quarter 3, Calendar Year 1993

Summary of Research Progress

This summary covers research activities for the cited contract during the third quarter of CY 1993. It discusses work in two major areas:

a. Optimization of systems described by simulations, with application to manufacturing problems.

b. Measuring value of information, with application to system testing.

a. Optimization of systems described by simulations

Area Overview

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow, particularly so when there are constraints on the allowable values of the parameters (for example, upper or lower bounds). Methods based on stochastic approximation may also experience difficulties if the function being optimized is not differentiable at certain points.

A major focus of work under this contract has been the application to simulation optimization of modern algorithms from nonsmooth analysis (specifically, methods of the bundle type), often in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination

methods are intended to replace the method of stochastic approximation, which is the standard method used in this area to date.

A very significant advantage of the bundle-type methods is that they can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause severe difficulty for methods of stochastic approximation. Therefore, development and application of these methods should significantly expand the applicability and usefulness of this kind of stochastic optimization. This work is being done in cooperation with the research group of Professor Rajan Suri at the University of Wisconsin–Madison.

Progress

Two major accomplishments occurred in this area during the quarter ending 30 September 1993. The first was the presentation at the 1993 Summer Computer Simulation Conference (Boston, 19–21 July 1993) of a paper explaining the application of this methodology to a specific manufacturing problem (maximizing steady-state throughput in tandem production lines). This paper was simultaneously published in the proceedings of that meeting (E. L. Plambeck, B.–R. Fu, S. M. Robinson, and R. Suri, "Throughput optimization in tandem production lines via nonsmooth programming," in: J. Schoen, ed., *Proceedings of the 1993 Summer Computer Simulation Conference*, Society for Computer Simulation, San Diego, CA 1993).

The second principal accomplishment in this area was the completion and submission for publication in *Mathematical Programming* of a long paper presenting a complete description and mathematical justification of the methodology, along with substantial computational results in two application areas (stochastic PERT networks and tandem production lines). A copy of this paper is attached as Enclosure 1. Part of the work in this paper represented new progress in the subarea of stochastic scheduling, a hard and important area with many applications in such areas as manufacturing, project management, etc., as mentioend in last quarter's report.

b. Measuring value of information

Area Overview

This area involves integrating stochastic optimization into decision analysis, and using the resulting set of tools to measure the value of information that could be obtained from projects or activities proposed for performance (for example, testing of weapon systems). One of the ideas being investigated is to employ, in place of the conventional decision tree, a stochastic optimization model that handles the task of selecting all or part of an optimal decision. Use of such a model makes it possible to deal, in a decision analysis framework, with very complex optimization models involving hundreds or thousands of variables. Such situations could occur, for example, in modeling of proposed weapon systems where one wished to decide whether to deploy a proposed system or to conduct further testing.

Using the framework of Bayesian preposterior analysis along with information about the accuracy of the proposed tests and their cost, one can use simulations of the system's performance to estimate the value added by further testing and, consequently, to judge whether such testing is worthwhile.

Progress

Work continued this quarter on the problem, previously reported, of methodological support for a decision on whether to request a waiver by the Secretary of Defense on the legally required live fire testing of major systems. The law provides that a waiver may be granted provided that live fire testing would be "unreasonably expensive and impractical."

In August 1993 a short concept paper entitled, "Estimating expected value of system testing," was completed. A copy of this paper is attached as Enclosure 2. This paper will be considered for inclusion in a larger document on the issue of live fire testing, to be published by the U. S. Army Research Laboratory. Publication of the latter document is not expected until early 1994.

Quarterly Review

No quarterly review was held this quarter. A review is to take place in December 1993 and details on it will be included in the next quarterly report.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work on the two principal areas described above (optimization of simulations, with applications to manufacturing problems, and estimating the value of information). One of the principal activities will be revising and resubmitting to *Management Science* a paper entitled, "Convergence of subdifferentials under strong stochastic convexity," which justifies part of the simulation optimization methodology already described above. This paper was refereed, and the revision will respond to the referees' comments. We also plan to work on a path-following method for nonsmooth optimization, which is part of the theoretical and computational foundation work that supports the more applied portion of our research program.

Quarter 4, Calendar Year 1993

Summary of Research Progress

This summary covers research activities for the cited contract during the fourth quarter of CY 1993. It discusses work in two major areas:

a. Optimization of systems described by simulations, with application to manufacturing problems.

b. Measuring value of information, with application to system testing.

As work under this contract is nearly complete, the intensity of contract activity during this quarter was lower than in most previous quarters. This trend is expected to continue in preparation for completion of the contract in March 1994.

a. Optimization of systems described by simulations

Area Overview

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow, particularly so when there are constraints on the allowable values of the parameters (for example, upper or lower bounds). Methods based on stochastic approximation may also experience difficulties if the function being optimized is not differentiable at certain points.

A major focus of work under this contract has been the application to simulation optimization of modern algorithms from nonsmooth analysis (specifically, methods of the bundle type), often in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination methods are intended to replace the method of stochastic approximation, which is the standard method used in this area to date.

A very significant advantage of the bundle-type methods is that they can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause severe difficulty for methods of stochastic approximation. Therefore, development and application of these methods should significantly expand the applicability and usefulness of this kind of stochastic optimization. This work is being done in cooperation with the research group of Professor Rajan Suri at the University of Wisconsin–Madison.

Progress

Work continued in this area during the quarter ending 31 December 1993. As mentioned in last quarter's report, a long paper presenting a complete description and mathematical justification of our proposed methodology for simulation optimization, along with substantial computational results in two application areas (stochastic PERT networks and tandem production lines), was submitted for publication in *Mathematical Programming.* We received a set of referee reports on this paper near the end of this quarter; these reports were generally favorable, and the editor invited us to submit a revised version. Work on that version began at the very end of the quarter, and it is expected to comprise the major part of the contract activity during the next quarter.

Also during this quarter, we completed the revision of a paper entitled, "Convergence of subdifferentials under strong stochastic convexity," and resubmitted it to *Management Science.* This paper provides theoretical background for some of the computational work in simulation optimization that we have been doing. A copy of this paper is attached as Enclosure 2. We expect to hear of the editors' decision on it sometime during the next quarter.

b. Measuring value of information

Area Overview

This area involves integrating stochastic optimization into decision analysis, and using the resulting set of tools to measure the value of information that could be obtained from projects or activities proposed for performance (for example, testing of weapon systems). One of the ideas being investigated is to employ, in place of the conventional decision tree, a stochastic optimization model that handles the task of selecting all or part of an optimal decision. Use of such a model makes it possible to deal, in a decision analysis framework, with very complex optimization models involving hundreds or thousands of variables. Such situations could occur, for example, in modeling of proposed weapon systems where one wished to decide whether to deploy a proposed system or to conduct further testing.

Using the framework of Bayesian preposterior analysis along with information about the accuracy of the proposed tests and their cost, one can use simulations of the system's performance to estimate the value added by further testing and, consequently, to judge whether such testing is worthwhile.

Progress

Work continued this quarter on the problem, previously reported, of methodological support for a decision on whether to request a waiver by the Secretary of Defense on the legally required live fire testing of major systems. The law provides that a waiver may be granted provided that live fire testing would be "unreasonably expensive and impractical."

As previously reported, in August 1993 a short concept paper entitled, "Estimating expected value of system testing," was completed. During this quarter we learned that this paper had been accepted fo inclusion in a collection of work on the issue of live fire testing, to be published by the U. S. Army Research Laboratory. Publication of the latter document is not expected until early 1994.

Quarterly Review

A quarterly review was held on 16 December 1993 at Huntsville, AL by Mr. K. Pathak, COTR. A copy of the viewgraphs from this review is included as Enclosure 1.

Planned Activities for Next Quarter

During the coming quarter we expect to continue work on the optimization of simulations, with applications to manufacturing problems. The principal activity is expected to be revising and resubmitting to *Mathematical Programming* the paper on computational optimization of systems including production lines and project scheduling networks.

Quarter 1, Calendar Year 1994

Summary of Research Progress

This summary covers research activities for the cited contract during the first quarter of CY 1994. It discusses work in one area, namely optimization of systems described by simulations, with application to manufacturing problems.

Work under this contract was completed at the end of this quarter, with the contractually required person-hours having been delivered at the cost agreed upon.

Optimization of systems described by simulations

Area Overview

Many important applied problems involving system design and operation can be modeled analytically, but not in such a way as to be able to solve the resulting models in closed form. In particular, when the models involve uncertainty it is necessary in such cases to use simulation to explore their properties. Such simulations are very widely used in system design in manufacturing and in many other areas.

When the models involve parameters, simulation techniques will permit us to estimate system parameters (for example, throughput in a manufacturing line) for any fixed setting of the parameters. However, if the parameters are under our control then a natural question to ask is what is the *best* choice of those parameters to optimize whatever measure of performance we are using. Current simulation techniques do not answer this question well; for example, the best generally used method for parameter optimization in discrete-event simulations is stochastic approximation, which is often extremely slow, particularly so when there are constraints on the allowable values of the parameters (for example, upper or lower bounds). Methods based on stochastic approximation may also experience difficulties if the function being optimized is not differentiable at certain points.

A major focus of work under this contract has been the application to simulation optimization of modern algorithms from nonsmooth analysis (specifically, methods of the bundle type), often in combination with the well established technique of Infinitesimal Perturbation Analysis (IPA) for gradient estimation in simulations. These combination

methods are intended to replace the method of stochastic approximation, which is the standard method used in this area to date.

A very significant advantage of the bundle-type methods is that they can handle linear equality or inequality constraints, such as work allocation to groups of machines in a production line, or upper and lower bounds on certain variables, without difficulty. Such constraints can cause severe difficulty for methods of stochastic approximation. Therefore, development and application of these methods should significantly expand the applicability and usefulness of this kind of stochastic optimization. This work is being done in cooperation with the research group of Professor Rajan Suri at the University of Wisconsin–Madison.

Progress

Work continued in this area during the quarter ending 31 March 1994. As mentioned in last quarter's report, a long paper presenting a complete description and mathematical justification of our proposed methodology for simulation optimization, along with substantial computational results in two application areas (stochastic PERT networks and tandem production lines), had been reviewed for publication in *Mathematical Programming.* We received a set of referee reports on this paper near the end of the previous quarter (in December 1993); these reports were generally favorable, and the editor invited us to submit a revised version. Work on that version comprised the major part of the contract activity during the quarter covered by this report. At the end of the quarter the revision was nearly complete. (The paper was subsequently completed and resubmitted to the journal in May, 1994.)

Also during this quarter, we completed some preliminary research on the justification of the mathematical method used in the above paper. Justification of this method, called sample-path optimization, is expected to form the subject of a paper to be prepared in late Spring 1994. It will acknowledge support from this contract.

Quarterly Review

A quarterly review was held on 29 March 1994 at Huntsville, AL by Mr. K. Pathak, COTR. A copy of the viewgraphs from this review is included as Enclosure 1.